

# Chiral Symmetry Breaking, Trace Anomaly and Baryons in Hot and Dense Matter\*

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We propose an effective chiral Lagrangian with a chiral scalar introduced as a dilaton associated with broken conformal symmetry and responsible for the trace anomaly in QCD and discuss the properties of hadronic matter at high density and temperature. As the “dilaton limit” is taken, which drives a system from nuclear matter density to near chiral restoration density, a linear sigma model emerges from the highly non-linear structure. A striking prediction is that as the dilaton limit is approached, the omega-nucleon interaction gets strongly suppressed at high density. This is shown to be a firm statement at the quantum level protected by an infrared fixed point of the renormalization group equations derived in chiral perturbation theory.

PACS numbers: 21.30.Fe, 12.39.Fe, 21.65.Mn

## 1. Introduction

Non-perturbative aspects of QCD in low energies are expected to be captured using effective field theories constructed based on global symmetries of QCD Lagrangian and their breaking pattern. In the limit of massless quarks the Lagrangian possesses the chiral symmetry and scale invariance, both of which are dynamically broken in the physical vacuum due to the strong interaction. The QCD trace anomaly signals the emergence of a scale at the quantum level from the theory without any dimension-full parameters [1]. Thus spontaneous chiral symmetry breaking, which gives rise to a nucleon mass, and the trace anomaly are closely linked to each other [2] and dynamical scales in hadronic systems are considered to originate from them. How they behave under extreme conditions such as high temperature and density is one of the main issues in QCD [3].

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\* Presented at the HIC for FAIR Workshop and XXVIII Max Born Symposium, Wrocław, Poland, May 19-21 2011.

In nuclear physics, a scalar meson plays an essential role as known from Walecka model that works fairly well for phenomena near nuclear matter density [4]. On the other hand, at high density, the relevant Lagrangian that has correct symmetry is the linear sigma model, and the scalar needed there is the fourth component of the chiral four-vector  $(\vec{\pi}, \sigma)$ . Thus in order to probe highly hot/dense matter, we have to figure out how the chiral scalar at low temperature/density transmutes to the fourth component of the four-vector. In this contribution, we construct an effective chiral Lagrangian for hadrons implemented by the conformal invariance introducing a chiral scalar as a dilaton associated with broken conformal symmetry and responsible for the trace anomaly in QCD. As the “dilaton limit” [5] is taken, which drives a system from nuclear matter density to near chiral restoration density, a linear sigma model emerges from the highly non-linear structure with the omega meson decoupling from the nucleons [6]. We also show a conceivable link of the dilaton limit at quantum level to an infrared fixed point of the renormalization group equations formulated in chiral perturbation theory with the lowest-lying parity-doubled nucleons [7].

## 2. Role of Dilatons near Chiral Symmetry Restoration

The trace anomaly is implemented in a chiral Lagrangian by introducing a dilaton (or glueball) field representing the gluon condensate  $\langle G_{\mu\nu} G^{\mu\nu} \rangle$  [8]. Following [9], we write the trace anomaly in terms of “soft” dilaton  $\chi_s$  and “hard” dilaton  $\chi_h$ . The dilaton potential,

$$V(\chi) = V_s(\chi_s) + V_h(\chi_h), \quad (1)$$

is assumed to have a negligible mixing between soft and hard sectors in order to avoid an undesirably strong coupling of the glueball to pions. As suggested in [10], we will associate the soft dilaton with that component locked to the quark condensate  $\langle \bar{q}q \rangle$ . We assume that this is the component which “melts” across the chiral phase transition whereas the hard component remaining non-vanishing<sup>1</sup>. It was shown in [10] that the soft dilaton plays an important role in the emergence of a half-skyrmion phase at high density where a skyrmion turns into two half-skyrmions [12].

In introducing baryonic degrees of freedom, there are two alternative ways of assigning chirality to the nucleons. One is the “naive” assignment<sup>2</sup> and the other the mirror assignment. The “naive” assignment,

$$\psi_L \rightarrow L\psi_L, \quad \psi_R \rightarrow R\psi_R, \quad (2)$$

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<sup>1</sup> The “melting” of the soft component is observed in dynamical lattice calculation in temperature [11] but is an assumption in density.

<sup>2</sup> We put this terminology in a quotation mark since it is a misnomer, used merely to distinguish it from the alternative option.

is anchored on the standard chiral symmetry structure where the entire constituent quark or nucleon mass (in the chiral limit) is generated by spontaneous symmetry breaking. The alternative, mirror assignment [13, 14],

$$\begin{aligned}\psi_{1L} &\rightarrow L\psi_{1L}, & \psi_{1R} &\rightarrow R\psi_{1R}, \\ \psi_{2L} &\rightarrow R\psi_{2L}, & \psi_{2R} &\rightarrow L\psi_{2R},\end{aligned}\tag{3}$$

allows a chiral invariant mass term,

$$\mathcal{L}_m = m_0 (\bar{\psi}_2 \gamma_5 \psi_1 - \bar{\psi}_1 \gamma_5 \psi_2),\tag{4}$$

which remains non-zero at chiral restoration. This means that a part of the nucleon mass,  $m_0$ , must arise from a mechanism that is not associated with spontaneous chiral symmetry breaking. At present, analysis of various observables both in the vacuum such as pion-nucleon scattering etc. and in medium such as nuclear matter properties etc. based on linear and nonlinear sigma models with mirror baryons [15, 16] cannot rule out an  $m_0$  of a few hundred MeV. As one approaches the chiral restoration point, the two assignments, even if indistinguishable at low density/temperature, are expected to start showing their differences. The origin of such a mass  $m_0$  can be traced back to the non-vanishing gluon condensate in chiral symmetric phase and therefore the broken scale symmetry is possessed by the hard dilaton. In this way we attribute the origin of  $m_0$  to the hard component of the gluon condensate, which is chiral invariant [6].

What about the low-lying meson masses? Mended symmetry is the algebraic consequence of spontaneously broken chiral symmetry along with the assumption on the scattering amplitudes of Nambu-Goldstone bosons in large  $N_c$  and the mesons are assembled into a few of the irreducible representations [17]. A prediction of the mended symmetry near chiral symmetry restoration is that the pion and other lowest-lying mesons become massless and fill out a full representation of the chiral group. Generally, a chirally-invariant mass for the mesons, as introduced for the parity-doubled nucleons, is not excluded.

### 3. Dilaton Limit

Our aim is to derive an effective Lagrangian for the Nambu-Goldstone bosons, vector mesons and soft dilatons in the linear basis starting with the hidden local symmetric (HLS) Lagrangian following the strategy of Beane and van Kolck. The 2-flavored HLS Lagrangian is based on a  $G_{\text{global}} \times H_{\text{local}}$  symmetry, where  $G_{\text{global}} = [SU(2)_L \times SU(2)_R]_{\text{global}}$  is the chiral symmetry and  $H_{\text{local}} = [SU(2)_V]_{\text{local}}$  is the HLS [18]. The entire symmetry  $G_{\text{global}} \times H_{\text{local}}$  is spontaneously broken to a diagonal  $SU(2)_V$ . The basic quantities

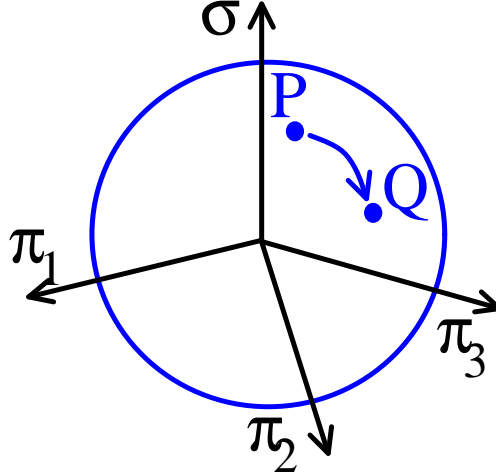


Fig. 1. A chiral sphere in  $(\vec{\pi}, \sigma)$  space.  $P$  on the sphere is mapped to another point  $Q$  via chiral transformations.

are the HLS gauge boson,  $V_\mu$ , and two matrix valued variables  $\xi_L$ ,  $\xi_R$ , which are combined in a  $2 \times 2$  special-unitary matrix  $U = \xi_L^\dagger \xi_R$ . Conformal invariance can be embedded in chiral Lagrangians by introducing a scalar field  $\tilde{\chi}$  via  $\chi = F_\chi \tilde{\chi}$  and  $\kappa = (F_\pi/F_\chi)^2$  [5].

Near chiral symmetry restoration the quarkonium component of the dilaton field becomes a scalar mode which forms with pions an  $O(4)$  quartet [5]. This can be formulated by making a transformation of a non-linear chiral Lagrangian to a linear basis exploiting the dilaton limit. Let  $\Phi$  be the basic building block of a linear sigma model  $\Phi = \sigma + i\vec{\tau} \cdot \vec{\pi}$  which transforms as  $\Phi \rightarrow L\Phi R^\dagger$ . Through chiral transformations a point on the 4-dimensional sphere is mapped to another point (see Fig. 1). One can also express  $\Phi$  in polar coordinates under the constraint  $F_\pi^2 = \sqrt{\sigma^2 + \vec{\pi}^2}$ , where a point is specified by three angles  $\vec{\theta} = (\theta_1, \theta_2, \theta_3)$ . Utilizing the polar decomposition the linear sigma model Lagrangian is rewritten to the standard non-linear chiral Lagrangian. In the following we show a linearized Lagrangian assuming two different chirality assignments to the positive and negative parity nucleons, the “naive” and mirror assignments.

In the “naive” model we introduce new fields,  $\Sigma$  and  $\mathcal{N}$ , as

$$\Sigma = \xi_L^\dagger \xi_R \chi \sqrt{\kappa} = s + i\vec{\tau} \cdot \vec{\pi}, \quad (5)$$

$$\mathcal{N} = \frac{1}{2} \left[ \left( \xi_R^\dagger + \xi_L^\dagger \right) + \gamma_5 \left( \xi_R^\dagger - \xi_L^\dagger \right) \right] N. \quad (6)$$

The linearized Lagrangian includes terms which generate singularities, neg-

ative powers of  $\text{tr} [\Sigma \Sigma^\dagger]$ , in chiral symmetric phase. Those terms carry the following factor:

$$X_N = g_V - g_A, \quad X_\chi = 1 - \kappa. \quad (7)$$

Assuming that nature disallows any singularities in the case considered, we require that they be absent in the Lagrangian, i.e.  $X_N = X_\chi = 0$ . We find  $\kappa = 1$  and  $g_A = g_V$ . A particular value,  $g_V = g_A = 1$ , recovers the large  $N_c$  algebraic sum rules [5]. Thus, we adopt the dilaton limit as

$$\kappa = g_A = g_V = 1. \quad (8)$$

The special value,  $g_V = 1$ , is in fact achieved as a fixed point of the renormalization group equations formulated in the chiral perturbation theory with HLS when one approaches chiral restoration from the low density or temperature side [7].

A noteworthy feature of the dilaton-limit Lagrangian is that the vector mesons decouple from the nucleons while their coupling to the Goldstone bosons remains. This has two striking new predictions. Taking the dilaton limit drives the Yukawa interaction to vanish as  $g_{VN}^2 = (g(1 - g_V))^2 \rightarrow 0$  for  $V = \rho, \omega$  for any finite value of the HLS gauge coupling  $g$ . In HLS for the meson sector, the model has the vector manifestation (VM) fixed point as one approaches chiral symmetry restoration, therefore the HLS coupling  $g$  also tends to zero proportional to the quark condensate. It thus follows that combined with the VM, the coupling  $g_{VN}$  will tend to vanish rapidly near the phase transition point. In nuclear forces, what is effective is the ratio  $g_{VN}^2/m_V^2$  which goes as  $(1 - g_V)^2$ . This means that (1) the two-body repulsion which holds two nucleons apart at short distance will be suppressed in dense medium and (2) the symmetry energy going as  $S_{\text{sym}} \propto g_{\rho N}^2$  will also get suppressed. As a principal consequence, the EoS at some high density approaching the dilaton limit will become softer *even without such exotic happenings as kaon condensation or strange quark matter*.

In the present scheme, the shortest-range component of the three-body forces also vanishes in the dilaton limit. The one-pion exchange three-body force involving a contact two-body force will also get suppressed as  $\sim g_{\omega N}^2$ . Thus only the longest-range two-pion exchange three-body forces will remain operative at large density in compact stars. How this intricate mechanism affects the EoS at high density is a challenging issue to resolve.

The dilaton limit is unchanged by the mirror baryons and therefore one arrives at similar phenomenological consequences to those mentioned above. The quenching of the short-range repulsion is independent of the chirality assignment of the nucleon and this is indicative of a universality of the short-distance interaction. How large is  $m_0$  at the chiral symmetry

restoration? It seems natural to expect that the source for non-zero  $m_0$  is in the hard dilaton condensate. A rough estimate can be made from thermodynamic considerations and the gluon condensate calculated on a lattice in the presence of dynamical quarks known to be [11]

$$\langle G_{\mu\nu} G^{\mu\nu} \rangle_{T_{\text{ch}}} \simeq \frac{1}{2} \langle G_{\mu\nu} G^{\mu\nu} \rangle_{T=0}, \quad (9)$$

at pseudo-critical temperature  $T_{\text{ch}} \sim 170$  MeV. Adopting the bag constant and mass for the hard dilaton as

$$B_h(T_{\text{ch}}) = \frac{1}{2} B(T=0), \quad m_{\chi_h}^2 = \frac{1}{2} m_G^2, \quad (10)$$

one finds  $m_0 = 210$  MeV. This is in agreement with the estimate made in vacuum phenomenology [7]. The nucleon in the mirror model stays massive at chiral symmetry restoration, so a different EoS from that in the “naive” model would be expected. This issue and more realistic estimate of  $m_0$  need to be carried out.

The axial-vector meson which figures in the mended symmetry can be dealt with on the same footing with the others and is introduced by generalizing  $H_{\text{local}}$  to  $G_{\text{local}}$  (GHLS) so that the entire symmetry of the theory becomes  $G_{\text{global}} \times G_{\text{local}}$  [18, 19]. Applying the same procedure as before, the non-linear GHLS Lagrangian with introducing a soft dilaton is transformed to its linearized form. One arrives at the vector and axial-vector meson masses proportional to the chiral order parameter. Thus, when chiral symmetry restoration takes place the mended symmetry becomes manifest. Introducing a chiral invariant mass for the mesons will modify the value of  $m_0$ .

Quantum loop corrections are systematically calculated in a chiral perturbation theory (ChPT) with HLS. In the “naive” assignment we assign the chiral counting  $\mathcal{O}(p)$  to the nucleon mass,

$$m_N \sim \mathcal{O}(p), \quad (11)$$

and the one-loop diagrams are evaluated in the relativistic formalism. In the mirror assignment the nucleon mass is not entirely generated by spontaneous chiral symmetry breaking and we identify the origin of the chiral invariant mass  $m_0$  with the explicit breaking of the QCD scale invariance, i.e. a hard dilaton, which has no direct link with the chiral dynamics. Consider  $m_0$  to be large compared with dynamically generated mass<sup>3</sup> and adopt a heavy

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<sup>3</sup> We associate  $m_0$  with the hard dilaton and thus the quantity  $m_{N_{\pm}} - m_0$  should *conceptually* be compatible to  $\Lambda_{\text{QCD}}$  even though  $m_0$  is a few hundred MeV as given above. This is changed e.g. in the presence of a scalar tetraquark state to be around 500 MeV [16].

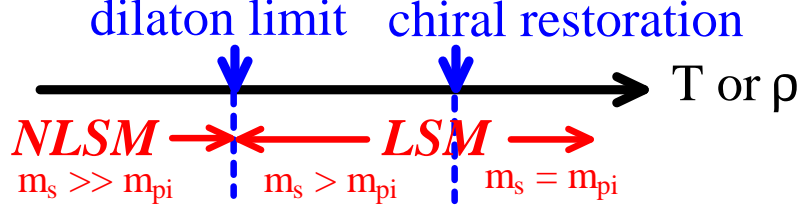


Fig. 2. Expected changeover of effective theories near chiral symmetry restoration.

baryon chiral perturbation theory (HBChPT) [20] in the presence of  $m_0$ . We write the nucleon momentum as

$$p^\mu = m_N v^\mu + k^\mu, \quad (12)$$

where  $v^\mu$  is the four-velocity with  $v^2 = 1$  and  $k^\mu$  is the residual momentum of order  $\Lambda_{\text{QCD}}$ , so that one can perform a chiral perturbation theory systematically in energy range below the chiral symmetry breaking scale  $\Lambda_\chi \sim 1$  GeV. A heavy-baryon doublet  $B$  is defined by

$$\begin{pmatrix} B_+ \\ B_- \end{pmatrix} = \exp[i m_0 v \cdot x] \begin{pmatrix} N_+ \\ N_- \end{pmatrix}. \quad (13)$$

Interestingly, the ChPT with either chirality assignment yields an infrared fixed point of the coupled renormalization group equations which can be identified with the dilaton limit [7].

#### 4. Conclusions and Remarks

We have shown how an effective theory near chiral symmetry restoration emerges from the dilaton-implemented HLS Lagrangian at the dilaton limit as illustrated in Fig. 2, and discussed its phenomenological implications at high baryon density. The soft dilaton is responsible for the spontaneous breaking of the scale symmetry and its condensate vanishes when the chiral symmetry is restored. In fact, topological stability of the half-skyrmion phase has been observed [12]. This is a strong indication that the configuration is robust and it could be associated with the scale symmetry restoration at high density in continuum theories.

One important prediction is that the repulsion at short distance in nuclear interactions should get suppressed at a density in the vicinity of the dilaton limit. Another hitherto unsuspected result is that the symmetry energy which plays a crucial role in the structure of compact stars also

should get suppressed. Put together, they will soften the EoS of compact-star matter at some high density. An interesting possibility is that our mechanism could accommodate an exotica-free nucleon-only EoS (such as AP4 in Fig. 3 of Ref. [21]) with a requisite softening at higher density that could be compatible with the  $1.97 \pm 0.04 M_\odot$  neutron star data [22]. It is an interesting and feasible phenomenological application of this model to determine the EoS and in-medium condensate of the dilaton as well as the onset of the dilaton limit at high density under a certain, e.g. mean field, approximation.

Nuclear structure studies tell us that the “hard core” is not a physical observable in medium, that is, it is not visible but shoved under what is known as “short-range correlation”. In fact, nuclear structure approaches anchored on effective field theory and renormalization group show that the “hard-core” repulsion present in two-nucleon potentials plays no role in low-energy physical observables [23]. Within the field theoretical framework we are working with, the short-distance repulsion is suppressed in the background or “vacuum” defined by density and the mended symmetry point of view would offer a possible way to understand it.

Our main observation on the suppressed repulsive interaction is a common feature in the two different assignments, “naive” and mirror, of chirality. Furthermore, the dilaton limit turns out to be an IR fixed-point of the renormalization group equations formulated in the chiral perturbation theory with HLS. Therefore decoupling of vector mesons from nucleons is a firm statement at quantum level. The derivative expansion in the HLS theory is justified for small gauge coupling,  $g \sim \mathcal{O}(p)$ , and in the limit of  $g \rightarrow 0$  the symmetry of the Lagrangian is in fact enlarged, which is known as “vector realization” of Georgi [24] and could protect the dilaton limit at quantum level. The nucleon mass near chiral symmetry restoration exhibits a striking difference in the two scenarios. How the dilaton-limit suppression of the repulsion – which seems to be universal independent of the assignments but may manifest itself differently in the two cases – will affect the EoS for compact stars is an interesting question to investigate.

In the scalar sector of low-mass hadrons, scalar quarkonium, tetra-quark states [25] and glueballs are expected to be all mixed. How this can happen has been studied in certain simple models, see e.g. [26] and references therein. It is an issue to be explored how the presence of the tetra-quark modifies the EoS. It is worth noting that using a toy model for constituent quarks and gluons implementing chiral and scale symmetry breaking a large  $m_\sigma \sim 1$  GeV in matter-free space is consistent with the lattice result regarding the thermal behavior of the gluon condensate [27], which is a conceivable scenario known from the vacuum phenomenology of the scalar mesons.



### Acknowledgments

I am grateful for fruitful collaboration with H. K. Lee, W.-G. Paeng and M. Rho. Partial support by the Hessian LOEWE initiative through the Helmholtz International Center for FAIR (HIC for FAIR) is acknowledged.

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